Spatially extended Unruh-DeWitt detectors for relativistic quantum information

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Unruh-DeWitt detectors interacting locally with a quantum field are systems under consideration for relativistic quantum information processing. In most works, the detectors are assumed to be point-like and therefore, couple with the same strength to all modes of the field spectrum. We propose the use of a more realistic detector model where the detector has a finite size conveniently tailored by a spatial profile. We design a spatial profile such that the detector, when inertial, naturally couples to a peaked distribution of Minkowski modes. In the uniformly accelerated case, the detector couples to a peaked distribution of Rindler modes. Such distributions are of special interest in the analysis of entanglement in non-inetial frames. We use our detector model to show that the noise detected in the Minkowski vacuum and in single particle states is a function of the detector's acceleration.

I. INTRODUCTION

Relativistic quantum information is a multifaceted area that incorporates key principles from quantum field theory, quantum information and quantum optics to answer, primarily, questions of an information theoretic nature. In order to implement quantum information tasks in relativistic settings it is necessary to find suitable localised systems to store information. Moving point-like detectors coupled to quantum fields have been considered to carry quantum information in spacetime [1–3], perform teleportation [4], and extract entanglement from the Minkowski vacuum [2, 5, 6]. Our research program aims at developing new detector models which are more realistic and simpler to treat mathematically so that they can be used in relativistic quantum information processing.

In this paper we propose the use of finite-size detectors [7–9] which are not only more realistic but also have the advantage of coupling to peaked distributions of modes. We design gaussian-type spatial profiles such that an uniformly accelerated detector naturally couples to peaked distributions of Rindler modes. By expanding the field in term of Unruh modes, we show that the accelerated detector couples simultaneously to two peaked distribution of modes corresponding to left and right Unruh modes. As expected, the same detector interacts with a gaussian distribution of Minkowski modes when it follows an inertial trajectory. In the Minkowski vacuum, the response of the detector has a thermal signature when it is uniformly accelerated and the temperature depends on the proper acceleration of the detector.

In the prototypical studies of quantum entanglement in non-inertial frames, observers are assumed to analyse states involving sharp frequency modes [10, 11]. In particular, recent works analysing the entanglement degra-

dation between global modes seen by uniformly accelerated observers consider states of modes labelled by Unruh frequencies [12–14]. Our analysis provides further insight into the physical interpretation of the particle states which were analyzed in these works. Finite-size detectors are suitable to discuss such results from an operational perspective since sharp frequency modes are an idealization of the peaked distributions finite-size detectors couple to. We show that the response of the finite-size detector when the state of the field has a single particle labelled by a Unruh frequency has a thermal term plus a correction with noise that depends on acceleration. Therefore, a degradation of global mode entanglement in non-inertial frames as a function of acceleration should be detected by uniformly accelerated finite-size detectors. Although global mode entanglement cannot be detected directly it can be extracted by Unruh-DeWitt type detectors becoming useful for quantum information tasks. Throughout our work we use natural units and the metric has signature $(-,+,+,+)$.

The action of a detector interacting with a quantum field in the interaction picture is given by [7, 8],

$$ S_I = \int dV \mathcal{M} \cdot \phi, \tag{1} $$

where dV is the volume element for the spacetime and $\mathcal{M}$ is the monopole moment of the detector parametrised by arbitrary coordinates in a $(3+1)$-dimensional spacetime. We will assume a flat spacetime and the field ϕ to be a real massive scalar field which satisfies the Klein-Gordon equation,

$$ \nabla_\mu \nabla^\mu \phi - m^2 \phi = 0, \tag{2} $$

where $m \geq 0$ is the free field mass and ∇_μ denotes the covariant derivative on the spacetime. We require the detector to have, in a comoving reference frame $(\tau, \vec{\zeta})$, a constant spatial profile. In this case the monopole moment of the detector factorizes into a temporal operator valued function and a spatial function $\mathcal{M}(\tau, \vec{\zeta}) =$

$M(\tau)f(\vec{\zeta})$ [15–17]. The temporal operator valued function describes the internal structure of the detector which we model by a two-level system of characteristic energy Δ therefore, $M(\tau) := \sigma_- e^{-i\Delta\tau} + \sigma_+^\dagger e^{i\Delta\tau}$. The operators σ_+ and σ_- create and annihilate, respectively, excitations in the internal structure of the detector. In this case, the action can be written in terms of the detector's proper time τ,

$$S_I = \int d\tau M(\tau)\tilde{\phi}(\tau), \tag{3}$$

where the field the detector couples to is given by,

$$\tilde{\phi}(\tau) := \int d^3\vec{\zeta}\sqrt{-g(\vec{\zeta})}f(\vec{\zeta})\phi(\tau,\vec{\zeta}) \tag{4}$$

and $\det(g_{\mu\nu}) = g$ is the determinant of the metric tensor. We assume that the center of the detector follows a classical trajectory in spacetime and the spatial profile $f(\vec{\zeta})$ determines how the detector couples to the field along the trajectory. This function, which must be real to ensure that the action is Hermitian, can be interpreted as a position dependent coupling strength. Consider $u_{\vec{\nu}}(\vec{\zeta}(\tau))$ to be field solutions to the Klein-Gordon equation evaluated along a point-like worldline parametrised by τ corresponding to the center point of the detector. The frequencies $\vec{\nu}$ of the modes are determined by an observer comoving with the center of the detector. The Hamiltonian in terms of these modes takes the form,

$$H_I = M(\tau) \cdot \int d^3\vec{\nu}\tilde{f}(\vec{\nu})\left(u_{\vec{\nu}}(\tau)a_{\vec{\nu}} + h.c.\right), \tag{5}$$

where $a_{\vec{\nu}}^\dagger$ and $a_{\vec{\nu}}$ are creation and annihilation operators associated to the field modes of frequency $\vec{\nu}$. The frequency distribution $\tilde{f}(\vec{\nu})$ corresponds to a transformation of $f(\vec{\zeta})$ into frequency space. In the ideal case where the detector is considered to be point-like, the spatial profile is $f(\vec{\zeta}) = \delta^3(\vec{\zeta} - \vec{\zeta}')$ (here $\delta^3(\vec{\zeta} - \vec{\zeta}') := \delta(\zeta_1 - \zeta_1')\delta(\zeta_2 - \zeta_2')\delta(\zeta_3 - \zeta_3')$ is the three dimensional Dirac delta distribution). The detector couples locally to the field and the coupling strength is uniformly equal for all frequency modes. When we model a finite-size detector, which corresponds to a more realistic situation, the detector couples naturally to a distribution of field modes. The frequency distribution will be determined by the spatial profile. In this sense the field $\tilde{\phi}(\tau)$ corresponds to a *window* of frequencies.

II. INERTIAL TRAJECTORY

Now we specify a trajectory for the detector. When the center of the detector follows an inertial trajectory it is convenient to use Minkowski coordinates $(t,\vec{x})$ where $\vec{x} := (x,y,z)$. In this case, the proper time of a comoving observer is $\tau = t$ and we can also write the comoving spatial coordinates as $\vec{\zeta} = \vec{x}$. The solutions to the Klein-Gordon equation correspond to plane waves,

$$u_{\vec{k}}(t,\vec{x}) := \frac{1}{\sqrt{2(2\pi)^3\omega}}e^{-i\omega t + i\vec{k}\cdot\vec{x}}, \tag{6}$$

where the frequency of the mode $\omega \equiv \sqrt{\vec{k}\cdot\vec{k} + m^2}$ is strictly positive and $\vec{k} \in \mathbb{R}^3$ denotes the momentum $\vec{k} := (k_x, k_y, k_z)$. The inner product satisfies $\left(u_{\vec{k}}, u_{\vec{k}'}\right) = \delta^3(\vec{k} - \vec{k}')$ [18]. In this case the creation and annihilation operators associated to the Minkowski field modes satisfy $\left[a_{\vec{k}}, a_{\vec{k}'}^\dagger\right] = \delta^3(\vec{k} - \vec{k}')$. The field can be expanded in Minkowski modes as

$$\phi(t,\vec{x}) = \int d^3\vec{k}\left(a_{\vec{k}}u_{\vec{k}}(t,\vec{x}) + h.c.\right) \tag{7}$$

From this, the frequency distribution expressed in Minkowski modes is,

$$\tilde{f}(\vec{k}) = \int d^3\vec{x}f(\vec{x})e^{+i\vec{k}\cdot\vec{x}}, \tag{8}$$

which is the Fourier transform of the spatial profile $f(\vec{x})$.

We now design a spatial profile tailored so that the corresponding frequency detection window of the detector is a gaussian distribution of modes peaked around a Minkowski frequency $\vec{\lambda}$. This choice is motivated by early works on relativistic entanglement where the states analysed involved sharp frequencies Ω and Ω'. For example, the Bell state,

$$|\phi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_\Omega|0\rangle_{\Omega'} + |1\rangle_\Omega|1\rangle_{\Omega'}\right), \tag{9}$$

which was analysed in a flat (1+1)-dimensional space in the charged [13] and uncharged massless bosonic case [12]. Entanglement for Bell states in non-inertial frames was also discussed for Dirac fields [14, 19, 20]. Sharp frequency states, $|1\rangle_\Omega = a_\Omega^\dagger|0\rangle$, are an idealisation of our gaussian wave-packets of the form

$$|1_\lambda\rangle = \int dk\Phi(\lambda,k)a_k^\dagger|0\rangle, \tag{10}$$

where $\Phi(\lambda,k)$ is a gaussian distribution centred around λ [12]. Our detector model will be useful to investigate questions of entanglement in non-inertial frames from an operational perspective and extract entanglement for relativistic quantum information processing. An interesting question, which we intend to address in following work, is how much entanglement can be extracted by our detectors from field mode entangled states as a function of the detector's acceleration.

We find that a gaussian frequency window of width σ centred around frequency $\vec{\lambda}$ as shown in FIG.(1), can be engineered by choosing the following spatial profile,

$$f(\vec{x}) = n_\sigma e^{-\frac{1}{2}\sigma^{-2}\vec{x}\cdot\vec{x}}\left(e^{-i\vec{\lambda}\cdot\vec{x}} + e^{+i\vec{\lambda}\cdot\vec{x}}\right), \tag{11}$$

which corresponds to a gaussian multiplied by a superposition of planes waves of opposite momentum $\vec{\lambda}$. n_σ is a normalisation constant. The spatial profile is therefore transformed into the momentum distribution,

$$\tilde{f}(\vec{k}) = e^{-\frac{1}{2}\sigma^2(\vec{k}-\vec{\lambda})\cdot(\vec{k}-\vec{\lambda})} + e^{-\frac{1}{2}\sigma^2(\vec{k}+\vec{\lambda})\cdot(\vec{k}+\vec{\lambda})}. \tag{12}$$
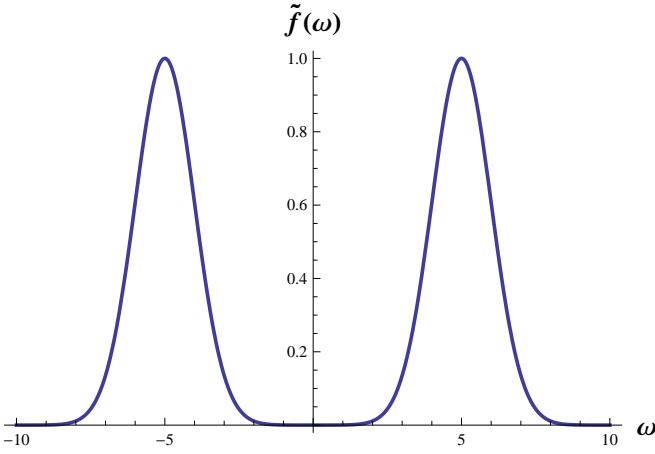
$$\tilde{f}(\omega)$$

FIG. 1: $1 + 1$ dimensional example of a frequency distribution peaked around $\pm\lambda$ given by Eq.(12) for $\sigma = 1$ and $\lambda = 5$. This frequency distribution peaks around around the desired frequency λ but has a double peaking due to the two exponential terms. In the $1 + 1$ massless case, the field is expanded as an integral over $\omega > 0$ and so the peak in the $\omega < 0$ region does not contribute.

This means that in order to couple our detector to a peaked gaussian distribution of modes centered around $\vec{\lambda}$ it is necessary to engineer a field-detector coupling strength which not only is peaked around the atom's trajectory but also oscillates with position. Sharp frequency modes $\tilde{f}(\vec{k}) = \delta^3(\vec{k} - \vec{\lambda}) + \delta^3(\vec{k} + \vec{\lambda})$ are obtained when $f(\vec{x}) \sim \exp(-i\vec{\lambda} \cdot \vec{x}) + \exp(+i\vec{\lambda} \cdot \vec{x})$. In the massless $1 + 1$-dimensional case the frequency distribution obtained from a given spatial profile is defined as a function of $\omega \geq 0$ only. Therefore, given the window profile peaks are sufficiently narrow and separated, the second peaking corresponding to the $\omega < 0$ region, does not contribute to the frequency window in this case. The field to which the detector couples given by Eq.(4) is therefore,

$$\phi(\tau) = \int d\omega N_\omega e^{-\frac{1}{2}\sigma^2(\lambda-\omega)^2} \left[e^{-i\omega t} a_\omega + e^{+i\omega t} a_\omega^\dagger \right] \quad (13)$$

In the general case, the frequency window is peaked around two modes corresponding to negative and positive momentum. Also see [21] which, complementary to this work, looks in detail at the coupling of Unruh-DeWitt detectors to Minkwoski modes. Eq.(12) shows that there is a trade-off between the width of the frequency window and the spatial profile of the detector.

We now proceed to calculate the instantaneous transition rate of the detector model we have introduced. The transition rate is a function of the detector's energy gap Δ [9] given by

$$\dot{F}_\tau(\Delta) := 2 \int_0^\infty ds Re\left[e^{-i\Delta s} W(\tau, \tau - s) \right], \quad (14)$$

where $W(\tau, \tau') := \langle \psi | \phi(\tau)\phi(\tau') | \psi \rangle$ is the so-called Wightman-function and $|\psi\rangle$ denotes the state of the field. Note that we have assumed that the detector is turned

on in the distant past. Expanding the field in terms of Minkowski modes we find that the vacuum transition rate for a stationary detector is

$$\dot{F}_\tau(\Delta) = -\Theta(-\Delta - m)\Xi(\Delta), \quad (15)$$

where

$$\Xi(\Delta) = \sqrt{\Delta^2 - m^2} |\tilde{f}(-\Delta)|^2 \quad (16)$$

and $\Theta(x)$ is the Heavisde theta function defined as

$$\Theta(x) = \begin{cases} 0 & : x < 0 \\ 1 & : x \geq 0 \end{cases} . \quad (17)$$

Note that in the above result we have explicitly assumed that $\tilde{f}(\vec{k}) = \tilde{f}(|\vec{k}|)$ i.e. the Fourier transform of the spatial profile $f(\vec{x})$ depends on the magnitude of $\vec{k}$ only. The result is explicitly independent of the time parameter τ. This can be traced back to the fact that an inertial trajectory is a stationary orbit in flat spacetime. The interesting result here is that the transition rate of the detector is tempered by the square of the frequency distribution $\tilde{f}$. We are also interested in analyzing the response of the detector when the field has a single Minkowski excitation,

$$|1_\Phi\rangle := \int d^3\vec{k} \Phi(\vec{k}) a_{\vec{k}}^\dagger |0\rangle, \quad (18)$$

where we define a delta normalised state to have the property $\int d^3\vec{k}|\Phi(\vec{k})|^2 = \delta^3(0)$ and a properly normalised state to have the property $\int d^3\vec{k}|\Phi(\vec{k})|^2 = 1$. This state is the generalization of Eq.(10) to three spatial dimensions. The Wightman-function in this case is

$$W(\tau, \tau') := \langle 1_\Phi | \phi(\tau)\phi(\tau') | 1_\Phi \rangle, \quad (19)$$

Writing the states and the field in terms of the Minkowski modes and normal ordering the associated operators we find that

$$\begin{aligned} W(\tau, \tau') = \int d^3\vec{k}|\Phi(\vec{k})|^2 \cdot \langle 0| \phi(\tau)\phi(\tau') |0\rangle \\ + 2\text{Re}\left[I_\Phi^*(\tau) I_\Phi(\tau') \right], \end{aligned} \quad (20)$$

where

$$I_\Phi(\tau) = \int d^3\vec{k} \Phi(\vec{k})\tilde{f}(\vec{k}) \frac{1}{\sqrt{\omega}} e^{-i\omega\tau}. \quad (21)$$

We notice that there are two terms in the Wightman-function. The first one corresponds to the vacuum state and the second is the contribution from the particle present in the field.

The single particle contribution factorizes into two independent functions of τ and τ'. This allows us to analyse the transition rate with relative ease. Substituting Eq. (20) into Eq. (14) we obtain

$$\iota_\tau(\Delta) := \int_0^\infty ds e^{-is\Delta} I_\Phi(\tau - s). \quad (22)$$

This integral is a Fourier transform of $I_\Phi(\tau - s)$ in the s variable and can be computed, either analytically or numerically, for specifically chosen $\tilde{f}$ and g. Employing the Riemann-Lebesgue lemma, which can only be used for functions which are integrable on the real line, one shows that $I_\Phi(\tau) \to 0$ as $\tau \to \pm\infty$ as long as $\tilde{f}$ and w are well-behaved. $I_\Phi(\tau)$ vanishes in the distant past and future where the detector is responding only to vacuum fluctuations. In other words, the detector only observes a constant spectrum in these asymptomatic regions. In the intermediate regions the oscillatory response is due to the presence of the particle.

III. ACCELERATED TRAJECTORY

We now consider a detector following an uniformly accelerated trajectory. Conformally flat Rinder coordinates $\xi = (\tau, \vec{\xi}) = (\rho, \xi, y, z)$ are a convenient choice in this case. The transformation between Rindler and Minkowski coordinates is given by

$$
\begin{aligned}
t &= a^{-1}e^{a\xi}\sinh(a\rho) \\
x &= a^{-1}e^{a\xi}\cosh(a\rho) \\
y &= y \\
z &= z
\end{aligned}, \tag{23}
$$

where a is a postive parameter. This transformation holds for the spacetime region $|t| > x$ which is called right Rindler wedge. The coordinate transformation for $|t| > -x$ (left region) differs from Eq. (23) only by an overall sign in the x coordinate. The coordinates are tailored specifically to the trajectory $\vec{\xi} = \vec{0}$ so that an observer travelling along this worldline will measure a proper acceleration $\sqrt{-A^\mu A_\mu} = a$ and the proper time is parametrised by the coordinate time ρ. The Klein-Gordon equation for a massive bosonic field in a (3+1)-dimensional flat spacetime in this case takes the form

$$
\partial_{\rho\rho}\phi - \left[\partial_{\xi\xi} + e^{2a\xi}(\partial_{yy} + \partial_{zz}) - m^2 e^{2a\xi}\right]\phi = 0, \tag{24}
$$

and the solutions are the Rindler modes [12, 18]

$$
\begin{aligned}
u_{\Omega,\vec{k}_\perp,\alpha}(\rho,\vec{\xi}) &:= N_{\Omega/a}K_{i\Omega/a}\left(Ma^{-1}e^{a\xi}\right)e^{-i\Lambda_\alpha(\rho,\vec{\xi})} \\
\Lambda_\alpha(\rho,\vec{\xi}) &:= \alpha\rho\Omega - \vec{k}_\perp \cdot \vec{x}_\perp,
\end{aligned} \tag{25}
$$

with $M = \sqrt{\vec{k}_\perp \cdot \vec{k}_\perp + m^2}$ and $N_{\Omega/a}$ is the mode normalisation constant. The functions $K_{i\Omega/a}(R)$ are modified Bessel functions of the second kind. Here $\vec{x}_\perp := (y, z)$ and $\vec{k}_\perp := (k_y, k_z)$ are position and momentum vectors perpendicular to the direction of acceleration. Ω is strictly positive and denotes the dimensionless Rindler frequency and $\alpha = +1(-1)$ corresponds to right (left) Rindler regions, respectively. The canonical orthonormality relations for the $3 + 1$ massive field are,

$$
(u_{\Omega,\vec{k}_\perp,\alpha}, u_{\Omega',\vec{k}_\perp',\alpha'}) = \delta(\Omega - \Omega')\delta^2(\vec{k}_\perp - \vec{k}_\perp')\delta_{\alpha\alpha'}, \tag{26}
$$

and commutation relations satisfy

$$
[a_{\Omega,\vec{k}_\perp,\alpha}, a^\dagger_{\Omega'\vec{k}_\perp',\alpha'}] = \delta(\Omega - \Omega')\delta^2(\vec{k}_\perp - \vec{k}_\perp')\delta_{\alpha\alpha'}. \tag{27}
$$

From our coordinate definitions Eq. (23), we choose the detector to be travelling in the right Rindler wedge. This then implies that our comoving coordinates can be parametrised as $\tau = \rho$ and $\vec{\zeta} = \vec{\xi}$. The field expansion in terms of the parametrised Rindler modes is

$$
\phi(\tau,\vec{\xi}) = \int d\Omega d^2\vec{k}_\perp \left[u_{\Omega,\vec{k}_\perp,+}(\rho,\vec{\xi})a_{\Omega,\vec{k}_\perp,+} + \text{h.c.}\right] \tag{28}
$$

Note the left Rindler modes do not appear in Eq.28 as the detector is assumed to be moving in the right Rindler wedge. The explicit form of the accelerated detectors frequency distribution is

$$
\tilde{f}(\Omega,\vec{k}_\perp) = \int d^3\vec{\xi}e^{2a\xi}f(\vec{\xi})K_{i\Omega/a}\left(Ma^{-1}e^{a\xi}\right)e^{+i\vec{k}_\perp \cdot \vec{x}_\perp} \tag{29}
$$

The most significant difference between the inertial and the accelerated frequency distributions is the appearance of a non-trivial metric factor and the Bessel function. Note also that for both massless and massive fields, the Rindler modes are defined as an integral over $\Omega \in \mathbb{R}^+$, unlike the Minkwoski mode case. Eq.(29) is a Fourier transform in the y and z coordinates, however, it is a non-standard integral transformation in the ξ coordinate. Reminiscent of a Hankel transformation, we expect our desired properties of arbitrary mode peaking to still hold. Using the integral representation of the Modified Bessel function for the second kind

$$
K_{i\Omega/a}(R) = \frac{\sqrt{\pi}\left(\frac{1}{2}R\right)^{i\Omega/a}}{\Gamma(i\Omega/a + 1/2)}\int_0^\infty dt\frac{(\sinh(t))^{2i\Omega/a+1}}{e^{R\cosh(t)}}, \tag{30}
$$

valid for $\Omega/a > 0$ and $R > 0$, we can write the frequency distribution as a Fourier type integral that takes the form

$$
\tilde{f}(\vec{k}) = \int d^3\vec{\xi}\beta(\vec{\xi})e^{i\vec{\xi}\cdot\vec{k}}, \tag{31}
$$

where now $\vec{k} = (\Omega + \delta, k_y, k_z)$ and

$$
\beta(\vec{\xi}) = \sqrt{\pi}\frac{\left(\frac{1}{2}\frac{M}{a}\right)^{i\Omega/a}}{\Gamma(i\Omega/a + 1/2)}f(\vec{\xi})\int_0^\infty dr\frac{(\sinh(r))^{2i\Omega/a}}{e^{\frac{M}{a}e^{a\xi}\cosh(r)}} \tag{32}
$$

δ is a phase that is acquired from the integral representation of the modified Bessel function. This shows that, in principal, the standard properties of the Fourier transformation can be used to design a detector profile such that we obtain a peaked distribution in momentum space. For a concrete example, we shall consider the massless $1 + 1$ field case. The appropriate transformation, in terms of Rindler modes, is given by

$$
\tilde{f}(\Omega) = \int d\xi e^{2a\xi}f(\xi)e^{i\Omega\xi}. \tag{33}
$$

and the spatial profile we propose in this case is

$$f(\xi) = N(\sigma)e^{-2a\xi}e^{-\frac{1}{2}\sigma^{-2}\xi^2}\left(e^{-i\tilde{\lambda}\xi} + e^{+i\tilde{\lambda}\xi}\right) \quad (34)$$

which includes the conformal metric factor that arises from the Rindler coordinate transformation. Here $N(\sigma)$ is a normalization constant. This profile reduces the integral transformation (33) to a standard Fourier transformation and the resulting frequency distribution is

$$\tilde{f}(\Omega) = e^{-\frac{1}{2}\sigma^2(\tilde{\lambda}-\Omega)^2} + e^{-\frac{1}{2}\sigma^2(\tilde{\lambda}+\Omega)^2}. \quad (35)$$

Substituting this frequency distribution into Eq.(4), we obtain

$$\phi(\tau) = \int d\Omega N_{\Omega/a} e^{-\frac{1}{2}\sigma^2(\tilde{\lambda}-\Omega)^2}\left[e^{-i\Omega\tau}a_{\Omega,I} + h.c.\right] \quad (36)$$

In the limiting case where the acceleration is zero $\Omega \to \omega$ and the spatial profile reduces to the Minkowski profile given by Eq.(11).

We would now like to expand the field in terms of Unruh modes which pay an important role in the literature. These modes are given by [12, 18],

$$\begin{aligned}u_{\Omega,\vec{k}_\perp,R} &:= \cosh(r_{\Omega/a})u_{\Omega,\vec{k}_\perp,+} + \sinh(r_{\Omega/a})u^*_{\Omega,\vec{k}_\perp,-}\\u_{\Omega,\vec{k}_\perp,L} &:= \cosh(r_{\Omega/a})u_{\Omega,\vec{k}_\perp,-} + \sinh(r_{\Omega/a})u^*_{\Omega,\vec{k}_\perp,+}\end{aligned}, (37)$$

where $\tanh(r_{\Omega/a}) = e^{-\pi\Omega/a}$. Upon parametrising the modes with our accelerated comoving coordinates, i.e. $(\rho,\vec{\xi}) = (\tau,\vec{\xi})$, and noting that the left Rindler modes have no support in the right Rindler wedge, we find that the Unruh modes reduce to

$$\begin{aligned}u_{\Omega,\vec{k}_\perp,R}(\tau,\vec{\xi}) &= \cosh(r_{\Omega/a})u_{\Omega,\vec{k}_\perp,+}(\tau,\vec{\xi})\\u_{\Omega,\vec{k}_\perp,L}(\tau,\vec{\xi}) &= \sinh(r_{\Omega/a})u^*_{\Omega,\vec{k}_\perp,+}(\tau,\vec{\xi})\end{aligned}, (38)$$

Expanding the field in terms of Unruh modes on the right wedge results in Eq.(39) and the corresponding frequency window operator is given by Eq.(41).

$$\phi(\tau,\vec{\xi}) = \int d\Omega d^2\vec{k}_\perp N_{\Omega/a}K_{i\Omega/a}\left(\frac{M}{a}e^{a\xi}\right)\left[\cosh(r_{\Omega/a})e^{-i\Omega\tau+i\vec{k}_\perp\cdot\vec{x}_\perp}a_{\Omega,\vec{k}_\perp,R} + \sinh(r_{\Omega/a})e^{+i\Omega\tau-i\vec{k}_\perp\cdot\vec{x}_\perp}a_{\Omega,\vec{k}_\perp L} + h.c.\right] (39)$$

$$\phi(\tau) = \int d\Omega d^2\vec{k}_\perp N_{\Omega/a}\tilde{f}(\Omega,\vec{k}_\perp)\left[\cosh(r_{\Omega/a})e^{-i\Omega\tau}a_{\Omega,\vec{k}_\perp,R} + \sinh(r_{\Omega/a})e^{+i\Omega\tau}a_{\Omega,\vec{k}_\perp,L} + h.c.\right] (40)$$

In case the field is massless and in $1+1$-dimensions, the detector interacts with two peaked distributions corresponding to right and left Unruh modes respectively,

$$\phi(\tau) = \phi_R(\tau) + \phi_L(\tau) \quad (41)$$

where

$$\phi_R(\tau) = \int d\Omega N_{\Omega/a}e^{-\frac{1}{2}\sigma^2(\tilde{\lambda}-\Omega)^2}\left[\cosh(r_{\Omega/a})e^{-i\Omega\tau}a_{\Omega,R} + h.c.\right] \quad (42)$$

$$\phi_L(\tau) = \int d\Omega N_{\Omega/a}e^{-\frac{1}{2}\sigma^2(\tilde{\lambda}-\Omega)^2}\left[\sinh(r_{\Omega/a})e^{+i\Omega\tau}a_{\Omega,L} + h.c.\right] \quad (43)$$

Note that the effective interaction strength is now modulated by hyperbolic trigonometric functions. We are currently using this detector model to analyse entanglement extraction of sharp frequency Unruh states.

As uniform acceleration is also a stationary orbit of flat spacetime, we expect a time independent vacuum transition rate. Using the parametrised Unruh modes, we can calculate the transition rate of the accelerated detector. We find that for the field in its vacuum state,

$$\dot{F}_\tau(\Delta) = \frac{1}{e^{2\pi\Delta/a} - 1}\Xi(\Delta), \quad (44)$$

where

$$\Xi(\Delta) := \int d^2\vec{k}_\perp\left[N^2_{\tilde{\Delta}}|\tilde{f}(\vec{\Delta})|^2\Theta(\Delta)\right. \quad (45)$$
$$\left. - N^2_{-\tilde{\Delta}}|\tilde{f}(-\vec{\Delta})|^2\Theta(-\Delta)\right] \quad (46)$$

with $\pm\vec{\Delta} := (\pm\Delta,k_y,k_z)$ and $N_{\tilde{\Delta}}$ denotes the appropriate normalisation for the Rindler modes. We can see immediately that the transition rate of the detector is the expected thermal distribution, where the temperature is inversely proportional to the acceleration parameter a, but again modified by the frequency of the field operator. We also note that Eq. (44) satisfies the Kubo-Martin-Schwinger condition [22, 23]

$$\dot{F}_\tau(\Delta) = e^{-\frac{2\pi}{a}\Delta}\dot{F}_\tau(-\Delta). \tag{47}$$

The transition rate is, as expected, independent of time due to the stationarity of the trajectory and the invariance of the vacuum state. Now we shall analyse the response of our accelerated detector model when the field contains a single Unruh particle. In the literature, well analysed states of the field correspond to maximally entangled Bell states, see for example, [10, 24, 25]. These states contain both the vacuum and a single Unruh particle. Our starting point will again be the Wightman-function which, for the one particle state, takes the form

$$W(\tau,\tau') \coloneqq \langle 1_p|\phi(\tau)\phi(\tau')|1_p\rangle \tag{48}$$

where $|1_p\rangle$ is a one Unruh particle state defined as $|1_p\rangle \coloneqq \int d\Omega d^2\vec{k}_\perp \Phi(\Omega,\vec{k}_\perp)a^\dagger_{\Omega,\vec{k}_\perp,p}|0\rangle$. Continuing in the exact same fashion as the Minkowski one particle state we find

$$W(\tau,\tau') = \int d\Omega d^2\vec{k}_\perp|\Phi(\vec{k})|^2\cdot\langle 0|\phi(\tau)\phi(\tau')|0\rangle \\ +2\mathrm{Re}\left[I_p^*(\tau)I_p(\tau')\right], \tag{49}$$

where $I_p(\tau) \coloneqq \int d\Omega d^2\vec{k}_\perp \tilde{f}(\Omega,\vec{k}_\perp)\Phi(\Omega,\vec{k}_\perp)U_{\Omega,p}(\tau)$ with

$$U_{\Omega,p}(\tau) = N_{\Omega/a}\begin{cases} \cosh(r_{\Omega/a})e^{-i\Omega\tau} &: p=R \\ \sinh(r_{\Omega/a})e^{+i\Omega\tau} &: p=L \end{cases} \tag{50}$$

An Unruh particle has with it an associated wavefunction, defined as $\int d\Omega d^2\vec{k}_\perp \Phi(\Omega,\vec{k}_\perp)U_{\Omega,p}(t,x)$, which must follow an accelerated trajectory peaked around some specified centre in position space. These are, by definition, accelerated plane waves. Thus, the accelerated detector will probe the accelerated plane waves as they approach. As the particles wavefunction oscillates as a function of τ, the detector will observe different phases at different times. It is these oscillations that contribute to the undulatory behaviour of the detectors transition rate. For the Unruh state, the corresponding accelerated expression for Eq. 22 again has a time dependent oscillatory integral. It is clear for appropriately behaved functions $\tilde{f}$ and g the same analysis can be applied here

as for the Minkwoski particle. The Riemann-Lebesgue lemma can be used to show that in the asymptotic past and future, the response of the detector is the same as the vacuum and hence has a thermal signature. As with the Minkowski particle, the intermediate regions between past and future asymptotic times give rise to an oscillatory response function. In the limit of high acceleration, the second terms in Eq. (49) become negligible and the state tends to the maximally mixed state. Thus, the single particle state of the field is dominated by the vacuum fluctuations.

IV. CONCLUSIONS

We introduce a detector model which naturally couples to peaked frequency distributions of Minkowski, Unruh and Rindler modes. This detector model is suitable for studies of entanglement extraction in non-inertial frames. In the (3 + 1)-dimensional case, the frequency window of the detector peaks around positive and negative momentum inducing a double peaking. In the (1+1)-dimensional case, frequency distributions naturally peak around a single frequency. We obtain analytical results for the instantaneous transition rates of the detectors undergoing inertial and uniformly accelerated motion. In particular, the transition rate of the accelerated detector is the expected thermal distribution modified by a smearing function that arises from the detectors spatial profile. We also showed that the well studied single Unruh particle states produce an oscillatory response that is only thermal in the asymptotic past and future. Since for accelerated detectors thermal noise is observed in both the vacuum and the single Unruh particle state, entanglement is expected to be degraded by the Unruh effect for global Unruh modes. We also see that the degradation effects occur for properly normalised wave packets.

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